

## Pure Mathematics 1

|  | Title | Estimated hours |
| :---: | :---: | :---: |
| $\underline{1}$ | Algebra and functions |  |
| a | Algebraic expressions: basic algebraic manipulation, indices and surds | 4 |
| $\underline{\text { b }}$ | Quadratic functions: factorising, solving, graphs and discriminants | 4 |
| C | Equations: quadratic/linear simultaneous | 4 |
| d | Inequalities: linear and quadratic (including graphical solutions) | 5 |
| e | Graphs: cubic and reciprocal | 5 |
| $\underline{\text { f }}$ | Transformations: transforming graphs; $\mathrm{f}(x)$ notation | 5 |
| 2 | Trigonometry |  |
| a | Trigonometric ratios and graphs, and area of a triangle in the form $\frac{1}{2} a b \sin C$ | 6 |
| $\underline{b}$ | Radians (exact values), arcs and sectors | 4 |
| 3 | Coordinate geometry in the ( $\boldsymbol{x}, \boldsymbol{y}$ ) plane: Straight-line graphs, parallel/perpendicular, length and area problems | 6 |
| 4 | Differentiation |  |
| a | Definition, differentiating polynomials, second derivatives | 6 |
|  | Gradients, tangents and normals | 5 |
| 5 | Integration: Definition as opposite of differentiation, indefinite integrals of $x^{n}$ | 6 |
|  |  | 60 hours |

## Pure Mathematics 2

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Proof: Examples including proof by deduction, proof by exhaustion and disproof by counter-example | 4 |
| 2 | Algebra and functions: Algebraic division and the factor and the remainder theorems | 4 |
| 3 | Coordinate geometry in the $(x, y)$ plane: Circles: equation of a circle, geometric problems on a grid | 7 |
| 4 | Sequences and series |  |
| a | Recurrence and iterations | 3 |
| $\underline{b}$ | Arithmetic and geometric sequences and series (proofs of 'sum formulae') | 4 |
| c | Sigma notation | 2 |
| d | The binomial expansion | 7 |
| 5 | Exponentials and logarithms: Exponential functions and the laws of logarithms | 8 |
| 6 | Trigonometry: Trigonometric identities and equations | 10 |
| 7 | Differentiation: Maxima and minima | 4 |
| 8 | Integration |  |
| $\underline{\text { a }}$ | Definite integrals and areas under curves | 5 |
| $\underline{\text { b }}$ | The trapezium rule | 2 |
|  |  | 60 hours |

## Further Pure Mathematics 1

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| $1 \begin{array}{lll}1 & \\ & \underline{a} \\ & \underline{b} \\ & \\ & \\ & \\ & \end{array}$ | Complex numbers |  |
|  | Introduction of complex numbers, basic manipulation | 3 |
|  | Complex conjugate, division and solving polynomial equations | 5 |
|  | Argand diagrams | 2 |
|  | Modulus and argument | 4 |
| 2 | Roots of quadratic equations |  |
|  | Roots of polynomial equations | 4 |
|  | Formation of polynomial equations | 2 |
| 3 | Numerical solution of equations: |  |
|  | Numerical solution of equations | 4 |
|  | Newton-Raphson method | 2 |
| 4 | Coordinate systems |  |
|  | Equations of parabola and rectangular hyperbola and the focus-directrix properties of the parabola | 6 |
|  | Tangents and normals to the parabola and hyperbola | 4 |
| $5 \begin{array}{ll}5 \\ & \\ & \underline{a} \\ & \underline{b}\end{array}$ | Matrix algebra integration |  |
|  | Matrix addition, subtraction and multiplication | 3 |
|  | Inverse of $2 \times 2$ matrices | 3 |
| 6 | Transformations using Matrices: Linear transformations | 8 |
| 7 | Series: Sums of series | 4 |
| 8 | Proof: Proof by mathematical induction | 6 |
|  |  | 60 hours |

## A level Mathematics: Further Pure Mathematics 2

## Further Pure Mathematics 2

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Inequalities: Algebraic inequalities and inequations | 5 |
| 2 | Series: Method of differences | 4 |
| 3 | Further complex numbers |  |
| a | Know and use $z=r \mathrm{e}^{\mathrm{i} \theta}=r(\cos \theta+\mathrm{i} \sin \theta)$ | 3 |
| $\underline{b}$ | De Moivre's theorem | 5 |
| c | Loci | 3 |
| d | Elementary transformations from the $z$-plane to the $w$-plane | 5 |
| 4 | First order differential equations |  |
| a | Integrating factors to solve first order differential equations | 5 |
| $\underline{b}$ | Differential equations reducible by means of a given substitution | 3 |
| 5 | Second order differential equations |  |
| a | Second order differential equations of the form $a \frac{\mathrm{~d}^{2} y}{\mathrm{~d} x^{2}}+b \frac{\mathrm{~d} y}{\mathrm{~d} x}+c y=\mathrm{f}(x)$ | 6 |
| b | Differential equations reducible by means of a given substitution | 3 |
| 6 | Maclaurin and Taylor series |  |
| a | Maclaurin series | 5 |
| b | Derivation and use of Taylor series | 4 |
| 7 | Polar coordinates |  |
| a | Convert between Cartesian and polar and sketch $r(\theta)$ | 4 |
| $\underline{b}$ | Area enclosed by a polar curve | 5 |
|  |  | 60 hours |

## A level Mathematics: Pure Mathematics 3

## Further Pure Mathematics 3

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| $\begin{array}{lll}1 & \\ & \underline{a} \\ & \underline{b}\end{array}$ | Hyperbolic functions |  |
|  | $\sinh x, \cosh x, \tanh x$ and their inverses | 4 |
|  | Logarithmic forms of the inverse hyperbolic functions, solving equations involving hyperbolic functions | 4 |
| 2 | Further coordinate systems |  |
|  | Equations of the ellipse and hyperbola and their focus-directrix properties | 3 |
|  | Tangents and normals to the ellipse and hyperbola | 3 |
|  | Simple loci problems | 4 |
| 3 | Differentiation |  |
|  | Differentiate the hyperbolic functions | 3 |
|  | Differentiate inverse trigonometric and hyperbolic functions | 4 |
| 4 | Integration |  |
|  | Integration of the hyperbolic and inverse hyperbolic functions | 3 |
|  | Integrate using hyperbolic and trigonometric substitutions | 3 |
|  | Reduction formulae | 4 |
|  | The calculation of arc length | 2 |
|  | The calculation of the area of a surface of revolution | 2 |
| 5 | Vectors |  |
|  | The vector product $\mathbf{a} \times \mathbf{b}$ and the scalar triple product $\mathbf{a} . \mathbf{b} \times \mathbf{c}$, and their applications | 3 |
|  | Problems involving points, lines and planes | 5 |
|  | Vector and Cartesian equations of a line and a plane | 3 |
| 6 | Further matrix algebra |  |
| abc | Linear transformations | 2 |
|  | Inverse of and $3 \times 3$ matrices | 2 |
|  | Eigenvalues and eigenvectors of $2 \times 2$ and $3 \times 3$ matrices | 4 |
| d | Reduction of symmetric matrices to diagonal form | 2 |
|  |  | 60 hours |

## A level Mathematics: Mechanics 1

## Mechanics 1

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Quantities and units in mechanics: Introduction to mathematical modelling and standard S.I. units of length, time and mass | 1 |
| 2 | Vectors in mechanics |  |
|  | Definitions, magnitude/direction, addition and scalar multiplication | 7 |
|  | Position vectors, distance between two points, application of vectors to displacement, velocity, acceleration and forces | 7 |
| $3 \begin{array}{ll} \\ & \\ & \underline{a} \\ & \underline{b}\end{array}$ | Kinematics of a particle moving in a straight line |  |
|  | Graphical representation of velocity, acceleration and displacement | 5 |
|  | Motion in a straight line under constant acceleration; suvat formulae for constant acceleration; Vertical motion under gravity | 6 |
| cd | Forces and Newton's laws |  |
|  | Newton's first law, Newton's third law, force diagrams | 3 |
|  | Newton's second law, ' $\mathrm{F}=\mathrm{ma}$ ', resolving forces, connected particles, problems involving smooth pulleys | 8 |
|  | Momentum and impulse; derivation of units and formulae Impulse-momentum principle. Conservation of momentum applied to collisions and 'jerking' string problems | 8 |
|  | Friction forces (including coefficient of friction $\mu$ ) | 4 |
| 5 | Statics of a particle: Equilibrium, Forces in vector form, Maximum value of the frictional force | 4 |
| 6 | Moments: Forces' turning effects | 7 |
|  |  | 60 hours |

## A level Mathematics: Mechanics 2

## Mechanics 2

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Kinematics of a particle moving in a straight line or plane |  |
| $\underline{\square}$ | Motion in a vertical plane under gravity; projectiles | 6 |
|  | Variable acceleration (use of calculus and finding vectors $\dot{\boldsymbol{r}}$ and $\ddot{\boldsymbol{r}}$ at a given time) | 6 |
| $\begin{array}{rrr}2 & \\ & \underline{a} \\ & \underline{b} \\ & \\ & \end{array}$ | Centres of mass |  |
|  | Centre of mass of a discrete mass distribution in one or two dimensions, framework and uniform lamina (rectilinear shapes) | 5 |
|  | Centre of mass of triangular, circular-based and composite laminas and centre of mass of a uniform circular arc | 5 |
|  | Modelling equilibrium: hanging bodies and systems free to rotate (about a fixed horizontal axis) | 4 |
| $\underline{\mathrm{b}}$$\underline{\text { c }}$ | Work and energy |  |
|  | Work and kinetic energy; derivation of units and formulae | 4 |
|  | Potential energy, work-energy principle, conservation of mechanical energy, problem solving | 6 |
|  | Power; derivation of units and formula | 4 |
| a$\underline{\mathrm{b}}$c | Collisions |  |
|  | Momentum as a vector (i, $\mathbf{j}$ problems); Impulse-momentum principle in vector form | 4 |
|  | Direct impact of elastic spheres. Newton's law of restitution. Loss of kinetic energy due to impact | 6 |
|  | Problem solving (including 'successive' impacts) | 4 |
| 5 | Statics of rigid bodies: Equilibrium and statics (including ladder problems) | 6 |
|  |  | 60 hours |

## A level Further Mathematics: Mechanics 3

## Mechanics 3

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Further kinematics: Motion in a straight line when the acceleration is a function of the displacement $(x)$ or time ( $t$ ); Setting up and solving differential equations | 6 |
| 2 | Elastic strings and springs and elastic energy |  |
|  | Hooke's law and definition of modulus of elasticity. Derivation of elastic potential energy formula | 6 |
|  | Problem solving: equilibrium and using the work-energy principle | 5 |
| 3 | Further dynamics |  |
| a | Particle moving in straight line with variable applied force; Using $F=m a$ to set up differential equations and solving | 6 |
| $\underline{b}$ | Newton's law of gravitation | 4 |
| c | Simple harmonic motion | 5 |
| 4 | Motion in a circle |  |
| $\underline{1}$ | Angular speed, central force, radial acceleration | 3 |
| $\underline{b}$ | Uniform motion in a horizontal circle | 6 |
| c | Motion in a vertical circle | 6 |
| $\begin{array}{rrr}5 & \\ & \underline{\mathrm{a}} \\ & \underline{\mathrm{b}} \\ & \underline{c} \\ & \\ & \underline{c}\end{array}$ | Statics of rigid bodies |  |
|  | Centre of mass of uniform rod, lamina, 3D rigid body using integration (and symmetry); Deriving formulae in formula book | 5 |
|  | Centre of mass of composite bodies; Simple cases of equilibrium of rigid bodies. | 4 |
|  | Conditions for toppling/sliding | 4 |
|  |  | 60 hours |

## Statistics 1

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| a$\underline{b}$ | Representation and summary of data |  |
|  | Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding | 5 |
|  | Use statistical diagrams for single-variable data to draw simple conclusions and to compare distributions; Understand and identify outliers; Understand and determine skewness | 8 |
| 2  <br>  $\underline{a}$ <br>  $\underline{b}$ | Probability: |  |
|  | Mutually exclusive events; Independent events | 4 |
|  | Using set notation for probability; Conditional probability | 6 |
| 3  <br>  a <br>  b | Correlation and regression |  |
|  | Scatter diagrams and least squares linear regression | 9 |
|  | The product moment correlation coefficient | 7 |
| $\begin{array}{rrr}4 & \\ & \underline{a} \\ & \underline{b}\end{array}$ | Discrete random variables |  |
|  | Use a discrete probability distribution to model simple situations; Identify the discrete uniform distribution | 6 |
|  | Mean and variance of discrete probability distributions | 7 |
| 5 | The Normal distribution: Understand and use the Normal distribution | 8 |
|  |  | 60 hours |

## Statistics 2

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| $\begin{array}{lll}1 & \\ & \text { a } \\ & \text { b } \\ & \mathrm{b} \\ & \text { c } \\ & \mathrm{c} \\ & \mathrm{d}\end{array}$ | The binomial and Poisson distributions |  |
|  | The binomial distribution | 5 |
|  | The Poisson distribution | 6 |
|  | Mean and variance of the binomial and Poisson distributions | 5 |
|  | Poisson distribution as an approximation to the binomial distribution | 5 |
| 2 | Continuous random variables |  |
|  | Continuous random variables, the probability density function and the cumulative distribution function | 9 |
|  | Summary statistics for continuous random variables | 6 |
| 3 | Continuous distributions |  |
|  | The continuous uniform distribution | 3 |
|  | Using the Normal distribution as an approximation to the binomial and Poisson distributions; Selecting the appropriate distribution | 7 |
| $\begin{array}{rrr}4 & \\ & \underline{\mathrm{a}} \\ & \\ & \underline{\mathrm{b}} \\ & \underline{c} \\ & \underline{\mathrm{c}} \\ & \underline{\text { d }}\end{array}$ | Hypothesis tests |  |
|  | Introduction to sampling terminology; Advantages and disadvantages of sampling | 3 |
|  | Language of hypothesis testing; Significance levels; Critical regions | 2 |
|  | Carry out hypothesis tests involving the binomial distribution | 5 |
|  | Hypothesis test for the mean of a Poisson distribution | 4 |
|  |  | 60 hours |

## Statistics 3

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Combinations of random variables: Distribution of linear combinations of independent Normal random variables | 7 |
| 2 | Statistical sampling: Understand and use sampling techniques; Compare sampling techniques in context | 4 |
| 3 | Estimation, confidence intervals and tests |  |
| a | Concepts of standard error, estimator and bias, including the quality of estimators | 5 |
| $\underline{b}$ | Concept of a confidence interval and its interpretation | 2 |
| C | Confidence interval for the mean of a Normal distribution with known variance | 3 |
| d | Statistical hypothesis testing for the mean of the Normal distribution | 6 |
| e | Use of the Central Limit Theorem | 4 |
| $\underline{\text { f }}$ | Hypothesis test for the difference between the means of two independent Normal distributions with known variances | 3 |
| g | Use of large sample results, hypothesis test for the difference between the means of two independent distributions with unknown variances | 3 |
| 4 | Goodness of fit and contingency tables: Chi-squared tests | 12 |
| 5 | Regression and correlation |  |
| $\underline{\text { a }}$ | Spearman's rank correlation coefficient | 5 |
| $\underline{b}$ | Hypothesis testing for zero correlation | 6 |
|  |  | 60 hours |

## Decision Mathematics 1

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Algorithms |  |
|  | Introduction to algorithms | 4 |
|  | Sorting, searching and packing algorithms | 8 |
| 2 | Algorithms on graphs |  |
|  | Introduction to graph theory | 2 |
|  | Minimum connectors (spanning trees) | 4 |
|  | Dijkstra's algorithm | 4 |
| 3 | Algorithms on graphs II |  |
|  | Route inspection problem | 4 |
|  | Travelling salesman problem | 8 |
| $\begin{array}{lll}4 & \\ & \underline{a} \\ & \underline{b} \\ & \underline{c} \\ & \underline{c} \\ & \underline{d}\end{array}$ | Critical path analysis |  |
|  | Activity networks; precedence tables | 5 |
|  | Critical path algorithm; earliest and latest event times | 4 |
|  | Total float; Gantt charts | 3 |
|  | Scheduling | 5 |
| $\begin{array}{rlr}5 & \\ & \underline{\text { a }} \\ & \underline{b} \\ & \underline{c}\end{array}$ | Linear programming |  |
|  | Formulation of problems | 3 |
|  | Graphical solutions | 4 |
|  | Integer solutions | 2 |
|  |  | 60 hours |

